PROCEEDINGS
OF THE
NATIONAL ACADEMY OF SCIENCES
Volume 7 NOVEMBER 15, 1921 Number 11

THE MOBILITIES OF ELECTRONS

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Communicated by R. A. Millikan, Aug. 17, 1921

In a recent paper[1] it was shown that the electrons could presumably travel through N_2 and H_2 gases in a free state at atmospheric pressure. It was found in some cases impossible to evaluate the mobilities of these electrons with the low frequencies of commutation available. The values of the mobilities indicated by these results appeared to be so much higher than the values obtained by earlier investigators that it was felt worth while to attempt to measure them accurately.

The mobilities of electrons were accordingly determined in N_2 using the high frequency oscillations obtained from two Western Electric Company "E" tube oscillators operating in parallel. The diagram of connections is shown in figure 1. In some cases the oscillations were taken directly from the primary condenser C_1, and at other times from the condenser C_2 of a secondary circuit tuned to resonance with the primary circuit. The method of measurement was the well known Rutherford alternating current method. The electrons used where photo electrons liberated by ultraviolet light from one of the plates. The ionization chamber used was similar to the one used in measurements of the constant of attachment of electrons to gas molecules,[2] except that still greater precautions were taken in this chamber to avoid contamination. The measurements were made in the same manner as were the measurements for the determination of the coefficient of attachment. The current to the electrometer plate was measured as a function of the value of the alternating potential between the plates. This current was then plotted against the potential difference thus yielding a mobility curve. The potential difference V_0, at which this curve cut the axis was substituted in the equation,

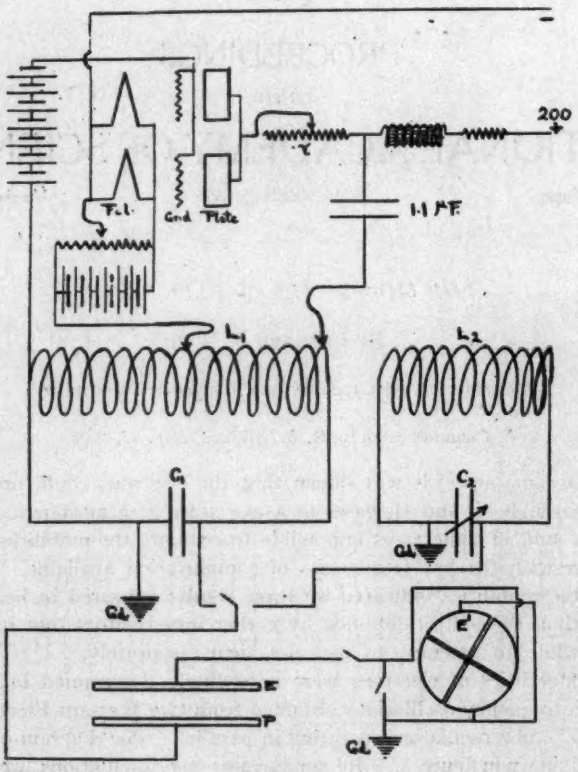

FIG. 1

$$U = \frac{\pi n d^2}{\sqrt{2V_0}},$$

in order to evaluate the mobility U. In this equation n is the frequency of the alternations, d is the distance between the plates in cm. and V_0 is the voltage in volts as read from the static voltmeter. In the oscillating circuits the potential was varied by varying the resistance r of the plate circuit. Since varying r altered the frequency n slightly the frequency of the oscillations was determined by means of a wave meter for several values of the voltage used in each determination. The experiments reported here covered a range of pressures from 600 mm. to 75 mm. The frequencies used varied from 7000 cycles per second to 150,000 cycles. The voltages used were varied from 10 volts to 300 volts and the plate distances lay between 2.0 cm. and 1.5 cm. The values of U thus obtained were reduced to give the mobility constant K at atmospheric

pressure by means of the equation, $K=(pU)/760$, where p is the pressure in mm. of mercury.

The mobilities found in this manner were astonishingly high. At 600 mm. mobilities of the order of magnitude of 15,000 cm. / sec. were obtained, while the highest previous mobility determined[3] in N_2 was 500 cm./sec. Furthermore the value of K obtained was not a constant as one would expect. K was found to be a function of the field strength and pressure. For a given pressure the value of K plotted as a function of V_0/d the field strength was found to lie on a hyperbola of the form, $K=\frac{a}{b+V/d}$. For different pressures a family of such hyperbolae were obtained which were expressed by the equation,

$$K=\frac{571{,}000}{21+760\,V_0/(pd)}.$$

This is shown in figure 2. The full curves are drawn from the above equation at values of p and d at which the measurements were made. The

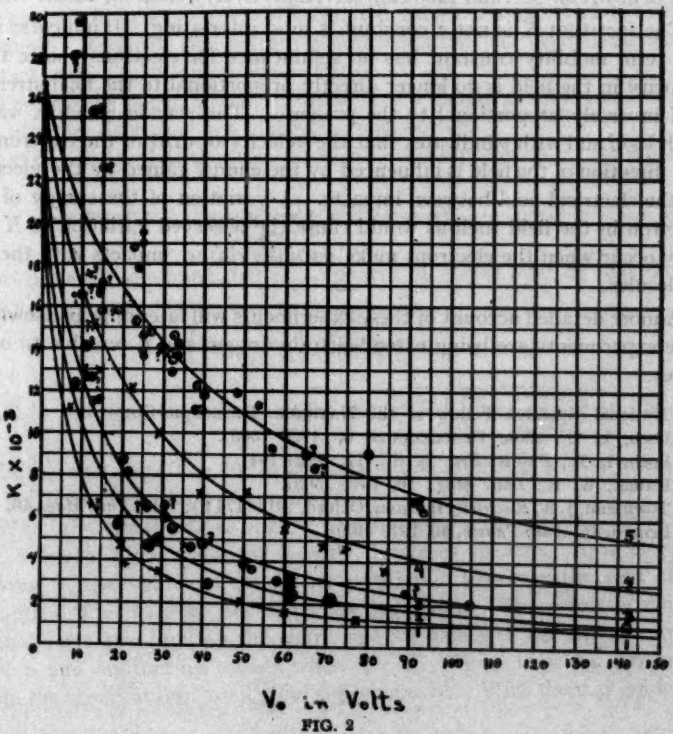

FIG. 2

points plotted are the actual experimental values of K obtained under the conditions of p, V_0, and d given. When the inaccuracies of such measurements are considered it is seen that the above equation represents the behavior of K to a satisfactory degree of approximation.

The success in measuring mobilities of such high values is probably due to the fact that especial care was taken to avoid contaminating gases, and to the fact that the frequencies employed were very high. The latter factor made it possible to measure the mobilities of purely electronic carriers only, for with the short intervals of time used only electrons which had made no attachments at all could succeed in crossing the plates. It seems likely that the low electronic mobilities observed by the previous workers were found for electrons which had been completely free for only a portion of their path between the plates because of the low frequencies of alternation used. The magnitude of the values obtained in these experiments is more nearly in accord with the values of electron mobilities predicted on the basis of the equations of Townsend[4] and Lenard[5] (i.e. of 6940 cm./sec. and 4260 cm./sec. respectively), than the earlier values.

The fact that K is not a constant is most interesting. It indicates that the term mobility constant has no significance for electrons; since their velocity in the field is no longer directly proportional to the field strength and inversely proportional to the pressure. The way in which K varies with V_0/d and with p indicates that the velocity of drift of the electrons in the direction of the field is influenced by the energy gained by the electron in the electrical field between impacts. A variation of the energy of the electron in the field such as would cause the observed variation of K can only occur when the electrons make *partially elastic* impacts with the gas molecules.

A more detailed account of these experiments will later appear elsewhere. The experiments are being extended to hydrogen and if possible to other gases.

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[1] Loeb, L. B., these PROCEEDINGS, 6, 1920 (335).

[2] Loeb, L. B., *Physic. Rev.* (N. S.), 17, 1921 (94).

[3] Haines, W. B., *Phil. Mag.*, 30, 1915 (503).

[4] Townsend, J. S., *Electricity in Gases*, Oxford, 1914 (174 ff); also *Phil. Mag.*, 40, 1920.

[5] Lenard, P., *Ann. Physik*, 40, 1913 (409).

A METHOD FOR OBTAINING CONSTANTS FOR FORMULAS OF ORGANIC GROWTH*

By H. S. Reed

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Communicated by R. Pearl, October 8, 1921

The growth of many organisms may be computed from the equation

$$\log \frac{x}{A-x} = K(t-t_1),$$

in which x represents the size of the organism at time t, A represents the size of the organism at maturity or at the end of a particular cycle of growth, t_1 is the time at which $x = A/2$, and K is a constant. This is the equation for an autocatalytic reaction and we are indebted to Robertson (1908) for showing its applicability to growth processes.

While the equation expresses the growth of plants with a high degree of accuracy, it often fails to fit the observed data in the early life of the organism. In other words, the computed values of x are too large when t is small. Recently this feature of the equation has been discussed by Mitscherlich (1919) and Rippel (1919).

The present paper intends to show how a simple graphic method may be used to overcome the difficulty just stated.

Let us assume that the true state of affairs is represented by the equation

$$\log \frac{x}{A-x} = K(t-t_1)^c.$$

This is merely assuming that the exponent of $(t-t_1)$ may or may not be unity. We may write

$$\log \left(\log \frac{x}{A-x} \right) = \log K + c \log (t-t_1)$$

This is the equation of a straight line if $\log \left(\log \frac{x}{A-x} \right)$ be used as ordinate and $\log (t-t_1)$ as abscissa. The intercept on the y-axis will be $\log K$ and the slope of the line c.

When $t < t_1$ the values of $(t-t_1)$ are negative in sign, as t approaches the value of t_1 the value of $(t-t_1)$ decreases to zero. As t increases beyond t_1 the values of $(t-t_1)$ are positive. However the sign of the quantity $(t-t_1)$ does not affect the logarithm. The result is that we actually get two lines on the chart, one for values of the equation when $t < t_1$ and another for values when $t > t_1$. These lines as determined from the observations, are seldom superimposed. With them as guides the

"best" straight line is drawn from which the values of the slope and intercept are read off. If the lines make much of an angle with each other it usually means that the value of t_1 has been poorly chosen. The choice of a value for t_1 is facilitated if one ascertains all the values of $\log\left(\log \frac{x}{A-x}\right)$, and from them determines by inspection the values of $(t-t_1)$ which will bring approximately equivalent values of $\log\left(\log \frac{x}{A-x}\right)$ as nearly as possible the same distance from the ordinate. For this purpose, the highest and lowest values of $\log\left(\log \frac{x}{A-x}\right)$ are less reliable than the intermediate values.

A few examples may be given.

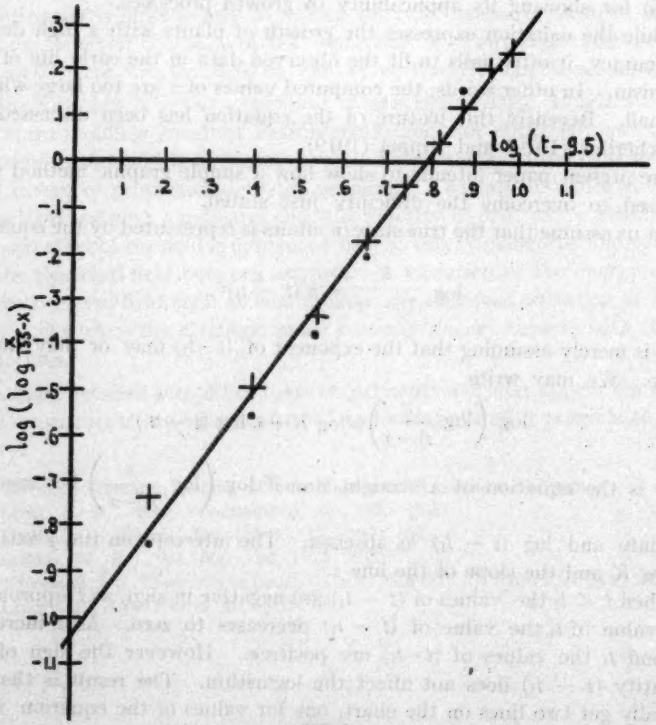

FIG. 1

Graph of values for lemon shoots. Points represent values of $\log (t-9.5)$ when $t < t_1$; crosses represent same when $t > t_1$.

The average length of 37 shoots on a group of lemon trees is given in the column headed x in table 1. The measurements were made each week for the first 21 weeks of their growth. From the observed length at intervals of a week the values of

$$\log\left(\log\frac{x}{135-x}\right) = \log K + c\log(t-9.5)$$

were ascertained and plotted (fig. 1). From the line thus obtained it was ascertained that $\log K = -1.05$ and $K = .0891$. The value of c as determined by the slope was 1.35.

Table 1 contains in the column headed x_1 the values computed from the equation

$$\log\frac{x}{135-x} = .0891(t-9.5)^{1.35}.$$

TABLE 1

GROWTH OF LEMON SHOOTS. COMPARISON OF OBSERVED AND CALCULATED VALUES

t WEEKS	x CM.	x_1 CM.	θ_1 CM.	x_2 CM.	θ_2 CM.
1	3.0	3.2	.2	5.3	2.3
2	4.9	5.7	.8	7.6	2.7
3	9.6	9.6	.0	10.8	1.2
4	15.6	12.8	−2.8	15.1	−0.5
5	25.8	23.4	−2.4	20.9	−4.9
6	37.3	33.4	−3.9	28.5	−8.8
7	46.8	44.8	−2.0	37.8	−9.0
8	56.5	55.6	−0.9	48.9	−7.6
9	65.2	61.4	−3.8	61.2	−4.0
10	73.5	73.6	.1	73.8	0.3
11	81.6	79.4	−2.2	86.1	4.5
12	91.4	90.2	−1.2	97.2	5.8
13	100.0	101.7	−1.7	106.5	6.5
14	110.9	111.6	.7	114.1	3.2
15	119.4	122.2	2.8	119.9	0.5
16	125.3	125.4	.1	124.2	−1.1
17	128.8	129.3	.5	127.4	−1.4
18	131.6	131.8	.2	129.7	−1.9
19	132.5	133.1	.6	131.4	−1.1
21	134.3	134.5	.2	133.2	−1.1
Root-mean-square deviation			1.88		4.40

The root-mean-square deviation of calculated from observed values was 1.88 cm.

In the column headed x_2 will be found values computed from the equation

$$\log\frac{x}{135-x} = .1635(t-9.5).$$

The root-mean-square deviation of these values is 4.40 cm.

TABLE 2

GROWTH OF SHOOTS ON YOUNG PEAR TREES

t DAYS	x CM.	x_1 CM.	θ_1 CM.	x_2 CM.	θ_2 CM.
0		6.8	6.8	8.1	8.1
7	5	10.0	5.0	10 7	5.7
14	11	14.6	3.6	15.4	4.4
21	19	20.7	1.7	21.3	2.3
28	29	28.6	−0.4	28.8	−0.2
35	40	39.2	−0.8	38.1	−1.9
42	48	49.8	1.8	48.5	0.5
49	59	59.0	0.	54.4	−4.6
55	69	67.6	−1.4	69.0	0.
63	76	79.1	3.1	80.4	4.4
70	86	88.2	2.2	88.8	2.8
77	94	95.5	1.5	95.6	1.6
84	100	101.5	1.5	100.9	0.9
91	102	105.5	3.5	104.8	2.8
98	108	108.5	0.5	107.6	−0.4
105	110	110.5	0.5	109.6	−0.4
112	111	111.8	0.8	110.9	−0.1
119	112	112.5	0.5	111.9	−0.1
126	113	113.0	0.	112.6	−0.4
Root-mean-square deviation			2.38		3.14

In a recent paper (Reed, 1920) I showed that the growth of shoots on Bartlett pear trees followed the course of an autocatalytic reaction very closely, except in the first few weeks of the season. During the early part of the growth period the calculated values were larger than those observed. Using the observed data I have computed values of K and c from the equation

$$\log\left(\log\frac{x}{114-x}\right) = \log K + c\log(t-47.4).$$

Plotting the values of this equation (fig. 2) the value of $\log K$ (the intercept) is -1.75 ($K=.0178$) and c (the slope) is 1.09.

When the size of the pear shoots is computed from the equation

$$\log\frac{x}{114-x} = .0178(t-47.4)^{1.09},$$

we obtain values agreeing more closely with the observed, as shown by table 2, where x = observed mean length of the young shoots at successive intervals; x_1 = length calculated from

$$\log\frac{x}{114-x} = .0178(t-47.4)^{1.09}.$$

θ_1 = deviations of x_1 from x; x_2 = length calculated from

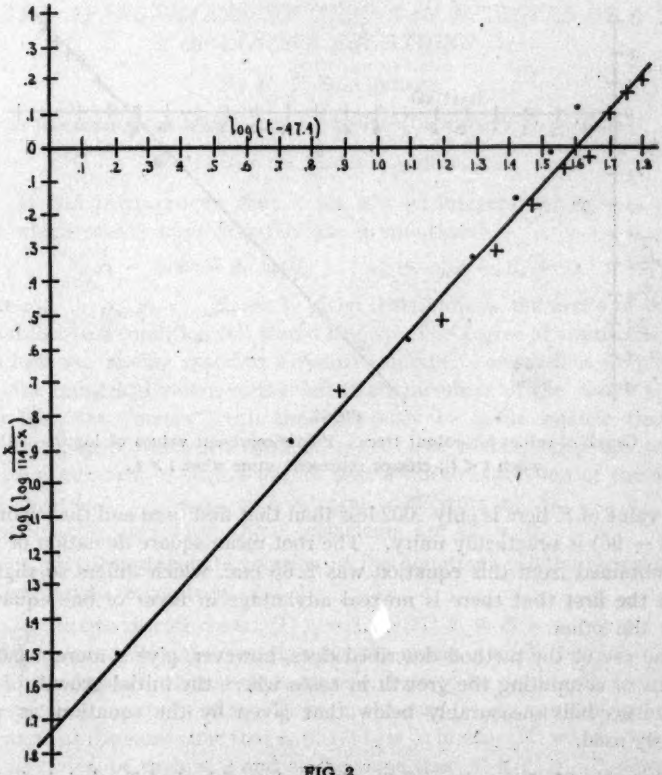

FIG. 2

Graph of values for pear shoots. Points represent values of log $(t-47.4)$ when $t < t_1$; crosses represent same when $t > t_1$.

$$\log \frac{x}{114 - x} = .0242(t - 47.4)$$

θ_2 = deviations of x_2 from x. The root-mean-square deviation of x_1 from x is 2.38 cm., and that of x_2 from x is 3.14.

Finally, a growth process which closely coincided with values computed from the usual formula, may be mentioned. In the paper cited (Reed, 1920) the average height of a population of young walnut trees (*J. regia*) was presented. The calculated values had a root-mean-square deviation from the observed of only 3.12 cm. The equation for growth as derived by graphic method was

$$\log \frac{x}{153 - x} = .0208(t - 96)^{1.03}.$$

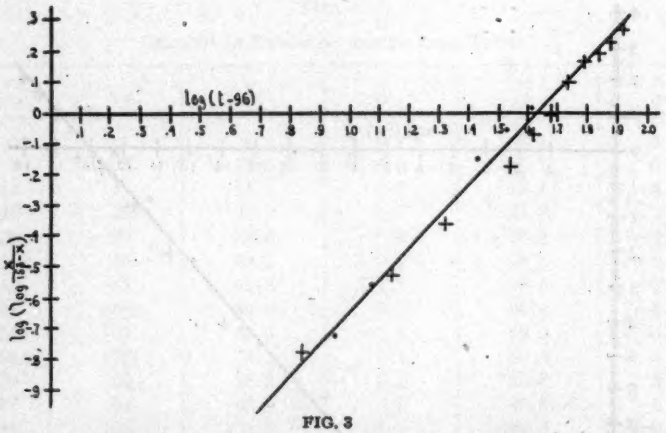

FIG. 3

Graph of values for walnut trees. Points represent values of log $(t - 96)$ when $t < t_1$; crosses represent same when $t > t_1$.

The value of K here is only .002 less than that first used and the exponent of $(t - 96)$ is practically unity. The root-mean-square deviation of values obtained from this equation was 2.65 cm., which differs so slightly from the first that there is no real advantage in favor of one equation over the other.

The use of the method described does, however, give a more accurate means of computing the growth in cases where the initial growth of the organisms falls measurably below that given by the equation as previously used.

* Paper No. 85, University of California, Graduate School of Tropical Agriculture and Citrus Experiment Station, Riverside, California.

Mitscherlich, E. A. (1919), "Ein Beitrag zum Gesetze des Pflanzenwachstums," *Fühling's Landw. Zeit.*, **68** (130–133).

Mitscherlich, E. A. (1919), "Zum Gesetze des Pflanzenwachstums," *Fühling's Landw. Zeit.*, **68** (419–426).

Reed, H. S. (1920), "The Nature of the Growth Rate," *J. Gen. Physiol.*, **2** (545–561).

Rippel, A. (1919), "Die Wachstumkurve der Pflanzen und ihre mathematische Behandlung durch Robertson und Mitscherlich," *Fühling's Landw. Zeit.*, **68** (201–214).

Robertson, T. B. (1908)," Further Remarks on the Normal Rate of Growth of an Individual and Its Biochemical Significance," *Arch. Entwicklugsmechn. Organ.*, **24** (108).

ON THE APPROXIMATE SOLUTIONS IN INTEGERS OF A SET OF LINEAR EQUATIONS

By H. F. Blichfeldt

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Read before the Academy, April 26, 1921

1. It has been proved that a set of $n+1$ integers[1] $x_1, x_2, \ldots, x_n, w$ exist which satisfy approximately the n equations[2]

$$x_1 - \alpha_1 w - \beta_1 = 0, \ldots, x_n - \alpha_n w - \beta_n = 0, \qquad (1)$$

where $\alpha_1, \ldots, \alpha_n, \beta_1, \ldots, \beta_n$ are $2n$ given real numbers, the first n of which are subject to a condition (C) stated below. The degree of approximation is as follows: having specified a positive number ϵ as small as we please, then the numerical values of the left-hand members of the above equations (i.e., the "errors") can simultaneously be made smaller than ϵ. The condition (C) imposed upon $\alpha_1, \alpha_2, \ldots. \alpha_n$ is this: they are to be irrational numbers of such a nature that a linear expression of the form $a_0 + a_1\alpha_1 + \ldots + a_n\alpha_n$, with integral coefficients $a_0, a_1, \ldots, a_n$, can vanish exactly *only* when these coefficients are all $= 0$.

For a given approximate solution of the equations (1) the errors shall be designated, respectively, by $\epsilon_1, \ldots, \epsilon_n$.

2. In the two special cases: (I) $n = 1$, or (II) $\beta_1 = \beta_2 = \ldots = \beta_n = 0$, we can make additional demands upon the errors, namely (and this is of importance in applications) we can cause them to be smaller than certain corresponding functions of w. Thus, in the case (I), we can demand that $\epsilon_1 < \epsilon$ and at the same time that $\epsilon_1 < 1/(4\,|\,w\,|)$; in case (II) we can demand that $\epsilon_1, \ldots, \epsilon_n$ be each $< \epsilon$ and at the same time $< k/(|\,w\,|)^m$, where $k = n/(n+1)$, and $m = -(n-1)/n$; furthermore, the condition (C) is not necessary in case (II).[3] These results may be stated in another way: in case (I), α being irrational, an infinite number of sets of integers $(x', w'), (x'', w''), \ldots.$ exist which, when substituted in turn in $x - \alpha w - \beta = 0$, will produce errors $\epsilon', \epsilon'', \ldots.$ of such magnitudes that $\epsilon' < 1/(4\,|\,w'\,|), \epsilon'' < 1/(4\,|\,w''\,|)$, etc. Similarly, in case (II) we have an infinite number of solutions satisfying corresponding inequalities.

3. In the general case the errors cannot be made to satisfy such extra demands. Stated more precisely: *no matter what function $f(w)$ be assumed, if only it be subjected to the conditions* (D) *below, we can always construct a set of equations* (1) *with attendant condition* (C), *which do not admit an infinite number of approximate solutions in integers $(x_1, x_2, \ldots, x_n), \ldots.$, if we demand the following degree of accuracy for each solution: $\epsilon_1 < f(|\,w\,|), \ldots, \epsilon_n < f(|\,w\,|)$. In fact, we may even substitute the weaker demand.*

$$\epsilon_2\epsilon_3 \ldots \epsilon_n + \epsilon_1\epsilon_3 \ldots \epsilon_n + \ldots + \epsilon_1\epsilon_2 \ldots \epsilon_{n-1} < f(|w|). \qquad (2)$$

The conditions (D) referred to are: for every positive integer w, $f(w)$ is a positive number which decreases when w increases, and approaches 0 as a limit when w tends to ∞.

4. We proceed to give an outline of the proof, limiting ourselves to the case $n = 3$, which is sufficiently illustrative. The function $f(w)$ being given subject to (D), we shall take as our equations the following set (under an obvious change of notation):

$$x - \alpha w - r\alpha = 0, \; y - \beta w - s\beta = 0, \; z - \gamma w - t\gamma = 0. \qquad (3)$$

Here r, s, t represent three arbitrary, but different integers, while α, β, γ are defined as the limits approached when j tends to ∞ by three series of generalized (Jacobian) continued fractions x_j/w_j, y_j/w_j, z_j/w_j; $j = 1, 2, \ldots$. We take at the outset any four sets of non-negative integers $(x_1, \ldots, w_1)$, $\ldots$, $(x_4, \ldots, w_4)$, such that the determinant $(x_1y_2z_3w_4) = 1$; succeeding sets $(x_j, \ldots, w_j; j > 4)$ are constructed from previous sets by the rule: $x_j = s_{j-1}x_{j-1} + x_{j-4}$, $y_j = s_{j-1}y_{j-1} + y_{j-4}$, etc., involving a series of positive integers $s_4, s_5, s_6, \ldots$, which, in turn, are expressed in terms of another series designated by [3], [4], [5], $\ldots$; namely, we put $s_j = 2^{[j]-[j-1]}$. The members of the new series are any set of positive integers satisfying the following conditions:

$[3] = 1$; when $j > 3$ take $[j] \geqq 3[j-1]$, $f(2^{[j]-[j-1]}) < 2^{-6[j-1]}$.

5. For convenience in the further development we make use of the letters M, m, μ to represent certain non-negative coefficients (constants or variables), the actual values of which are not required: M for a number having a lower bound > 0, m for one having an upper bound, and μ for one having both a lower bound > 0 and an upper bound.

We note the following preliminary relations:

$x_j = \mu.2^{[j-1]}$, $y_j = \mu.2^{[j-1]}$, etc.;

$\xi_j \equiv x_j - \alpha w_j = \pm m/s_j$, $\eta_j \equiv y_j - \beta w_j = \pm m/s_j$, $\zeta_j \equiv z_j - \gamma w_j = \pm m/s_j$; $x_iw_j - x_jw_i = (x_iw_j) = \pm m.2^{[j-1]-[i]+[i-1]}$ when $j > i$, $y_iw_j - y_jw_i = (y_iw_j) =$ etc., etc. Moreover, (x_iw_j), etc., do not vanish when j is taken sufficiently high.

(The condition (C), §1, is satisfied by α, β, γ. For, an equation $a_0 + a_1\alpha + a_2\beta + a_3\gamma = 0$ would imply $a_0w_j + a_1x_j + a_2y_j + a_3z_j = 0$ for every subscript j above a certain number. But this requires $a_0 = a_1 = a_2 = a_3 = 0$, since the determinant $(x_jy_{j+1}z_{j+2}w_{j+3}) = \pm 1$.)

6. If x, y, z, w represent any four integers, then at least two of the numbers

$$A = xw_j - x_j(w+r), \quad B = yw_j - y_j(w+s), \quad C = zw_j - z_j(w+t)$$

are of magnitude $\pm M.2^{[j-1]-2[j-2]}$, when j exceeds a certain number. For selecting any pair, say A, B, we have

$$(Aq - Bp)\,w_{j-1} + (r - s)pq \equiv 0 \pmod{w_j},$$

where $p=(x_jw_{j-1})$, $q=(y_jw_{j-1})$. But this congruence is found to imply $|Aq - Bp|w_{j-1} > wj$ for sufficiently high values of j. Hence, either $|Aq|$ or $|Bp|$ or both are $> w_j/w_{j-1}$.

7. Reverting now to the equations (3), we take any approximate solution in integers, say x, y, z, w. Assuming for our purpose that $|w| \geqq 2^{[4]-[3]}$, we determine the positive integer j such that

$$2^{[j-1]-[j-2]} \leqq |w| < 2^{[j]-[j-1]}.$$

We have

$$w_je_1 = |A+(w+r)\xi_j|, \quad w_je_2 = |B+(w+s)\eta_j|, \quad w_je_3 = |C+(w+t)\zeta_j|,$$

and it follows from the results above that at least one of the three products e_1e_2, e_2e_3, e_3e_1 is of magnitude $M.2^{-4[j-2]}$ for sufficiently large values of j. But, under the same condition,

$$M.2^{-4[j-2]} > 2^{-6[j-2]} > f(2^{[j-1]-[j-2]}) \geqq f(|w|).$$

Hence, when j exceeds a certain number, the condition (2) is not fulfilled. That is, the values of w satisfying this condition are limited.

[1] Throughout this paper the term "integer" means "a positive or negative integer or zero," when it is not specially defined.

[2] Kronecker, L., *Monatsberichte Kgl. Preuss. Aka. Wiss.*, 1884 (1179ff., 1271ff.); *Werke*, 3^I, Leipzig, 1899 (49–109).

[3] See Dickson, L. E., *History of the Theory of Numbers*, 2; *Carnegie Institution, Wash.*, 1920 (93–99), for references.

A PHYSICAL BASIS FOR EPIDEMIOLOGY

By Simon Flexner and Harold L. Amoss

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Read before the Academy, April 26, 1921

In the autumn of 1918 there swept through one of the mouse breeding rooms of the Rockefeller Institute a destructive epidemic of mouse typhoid—an infection of mice with a bacillus of the enteritidis group of organisms to which the name of *Bacillus typhi murium* has been given. The history of the epidemic is instructive: the original mice of the population, numbering about 3000 at the time of the epidemic, came from a breeder in Massachusetts and had been purchased some time before and moved *en masse* to the Rockefeller Institute. In the meantime many new mice had been born of this stock, and many of the original mice had died or been employed for experiment, so that only a small residue of the original population remained. In other words, the epidemic of mouse typhoid arose among chiefly a new stock of mice, the offspring of an old stock believed on good grounds to have passed through previous outbreaks of the disease.

There is still another reason for supposing that the epidemic arose from within and was not imported from without this stock. Besides the breed-

ing room for the population accruing from the purchased mice, a second breeding room not far away is maintained for mice bred at the Institute from perfectly healthy stock. This second breeding station has been developed from small beginnings and now has an average mouse population of 3000. The personnel in charge of each breeding room is distinct and does not mingle. During the period of many months in which the epidemic continued, rose and fell in the one room, no death from mouse typhoid occurred in the other.

Our attention has long been directed to the phenomena of the epidemic prevalence of disease, and since the early years of the founding of the Rockefeller Institute opportunity was presented to observe in succession epidemics of meningitis, poliomyelitis, influenza, and latterly of encephalitis. The waves of these diseases, which have swept over the world since 1904, have aroused profound interest in the facts of epidemiology.

We seized, therefore, upon this outbreak of mouse typhoid in order to study experimentally an epidemic disease among small laboratory animals which could be assembled and observed in fairly large numbers. The epidemic referred to has supplied the cultures of the *Bacillus typhi murium* for the experiment and certain data of a statistical nature with which to compare our experimental results. The healthy stock of mice also referred to provided an unexceptionable material with which to attempt the production of an intentional epidemic.

The conditions of the experiment were simple. A kind of mouse village was set up and an isolated room away from all other animals was selected in which shelves were erected and the cages placed. The latter may be taken for streets and houses in an ordinary village. Each cage contained five healthy mice. The original population, later increased from time to time by the introduction of fresh increments, was 100. The epidemic was started by feeding with a virulent culture of *Bacillus typhi murium* ten mice placed in two cages midway of the other cages. The incidental contact between the culture fed mice and the others was secured by the attendant who fed the animals and periodically cleaned the cages. A spot map was kept in order to follow the events.

The preliminary feeding experiments led to the death from mouse typhoid of eight of the ten hand-fed and seven contact mice distributed in as many cages. No epidemic in the real sense ensued. The fatal outbreak of mouse typhoid which arose partook of the nature of sporadic instances of the disease.

When the conditions as regards death from the infection had become stationary (at about the end of 30 days), a fresh addition of 200 mice contained in cages of five each was brought into the village. The effect of this addition was striking: after a lapse of about five days the new

mice began to die of mouse typhoid, and five to ten days later the old mice (original population) also began to succumb. The total losses in this outbreak were 70 per cent, and the number of cages involved all, or 100 per cent. The epidemic endured about four weeks and then subsided, although an occasional death still occurred at intervals.

When a second equilibrium had been established, another addition of normal or healthy stock was introduced into the village. The succession of events was about the same as that just described. However, the peak of the epidemic was less high, the total percentage of deaths being 5 per cent, although 100 per cent of the cages was again attacked. After an interval, another equilibrium was reached, when still another addition of normal mice brought about a recrudescence, passing as in the other examples from the healthy new population to the exposed old population, with however a total percentage of deaths below the others.

These fluctuations or movements of the epidemic consisting of a series of waves have been repeated now about ten times. The general character or type of the wave or curve is always the same, although the height or peak as determined by the percentage of mortality varies. In every instance the mortality rate among the old mice became finally about the same as among the new.

The experiments are followed in two main ways: according to the mortality, controlled of course by bacteriological examination of the animals succumbing, and according to the living carriers of the *Bacillus typhi murium* produced. The feces and the urine of living mice are tested for the bacillus, and in general it may be stated that the percentage of carriers stands in inverse ratio to the mortality. In other words, high death rate means a low carrier rate and *vice versa*. An obvious, although by no means safe, deduction from this observation would be that the carriers have all recovered from non-lethal infection and hence are rendered resistant by an acquired immunity, and that the production of carriers is nature's way of limiting the epidemic and of bringing about the state of equilibrium described.

The main results of the study presented may be viewed as the mere beginnings of an undertaking to follow artificially reproduced epidemics among animals, which not only should simulate the epidemics naturally or spontaneously arising, as we say, among men and animals, but which being controllable may be made to yield up some of the underlying conditions affecting the movements among those naturally occurring epidemic outbreaks which we represent graphically in the form of curves, but which in relation to cause and effect we still are so largely ignorant.

The impulse to use epidemic diseases among laboratory animals to help solve the problem of epidemics among man has been felt by other experimenters. In particular Topley[1] in London has independently made use

also of the mouse and a bacillus of the enteritidis group with which to conduct experiments, and the reports he has already published, in spite of quite fundamental differences of procedure, are readily comparable with those we have obtained.

As a matter of practical importance perhaps the epidemic diseases affecting the respiratory tract may be more important than those in which the portal of entry into the body of the infectious microbe is by way of the digestive tube. The latter class has indeed been subjected to far greater control by sanitary and other measures than the former. But in endeavoring to work out the underlying principles ultimately to be assembled into the "laws" of epidemics, information from many sources will be required. It is part of our object, however, to report later on experimental epidemiological studies now in progress with a disease of laboratory animals in which the infecting bacillus thrives on the respiratory mucous membrane. It is obvious that the two chief factors involved in epidemiological investigations, namely the host and the microbe, are both highly intricate. Any comprehensive study will include both, and to the extent to which it is enlightening will account for the effects of the fluctuation of one on the response of the other, the two conditions having been correlated providing the data on which our ultimate understanding of epidemics will come to rest.

[1] *J. Hygiene*, 19, 1921 (349).